Chapter 7

Section 1 Circular Motion

Preview

- Objectives
- Tangential Speed
- Centripetal Acceleration
- Centripetal Force
- Describing a Rotating System
Objectives

- **Solve** problems involving centripetal acceleration.

- **Solve** problems involving centripetal force.

- **Explain** how the apparent existence of an outward force in circular motion can be explained as inertia resisting the centripetal force.
Tangential Speed

The tangential speed \( v_t \) of an object in circular motion is the object’s speed along an imaginary line drawn tangent to the circular path.

Tangential speed depends on the distance from the object to the center of the circular path.

When the tangential speed is constant, the motion is described as uniform circular motion.
Chapter 7
Section 1 Circular Motion

Centripetal Acceleration

Click below to watch the Visual Concept.
Centripetal Acceleration

The acceleration of an object moving in a circular path and at constant speed is due to a change in direction.

An acceleration of this nature is called a centripetal acceleration.

\[
\text{CENTRIPETAL ACCELERATION}
\]

\[
a_c = \frac{v_t^2}{r}
\]

Centripetal acceleration = \frac{(\text{tangential speed})^2}{\text{radius of circular path}}
Centripetal Acceleration, continued

Å (a) As the particle moves from A to B, the direction of the particle’s velocity vector changes.

Å (b) For short time intervals, $\Delta v$ is directed toward the center of the circle.

Å Centripetal acceleration is always directed toward the center of a circle.
Centripetal Acceleration, *continued*

- You have seen that **centripetal acceleration** results from a **change in direction**.

- In circular motion, an acceleration due to a **change in speed** is called **tangential acceleration**.

- To understand the difference between centripetal and tangential acceleration, consider a car traveling in a circular track.
  - Because the car is moving in a circle, the car has a **centripetal** component of acceleration.
  - If the car’s speed changes, the car also has a **tangential** component of acceleration.
Consider a ball of mass $m$ that is being whirled in a horizontal circular path of radius $r$ with constant speed.

The force exerted by the string has horizontal and vertical components. The vertical component is equal and opposite to the gravitational force. Thus, the horizontal component is the net force.

This net force, which is directed toward the center of the circle, is a centripetal force.
Centripetal Force, *continued*  

Newton’s second law can be combined with the equation for centripetal acceleration to derive an equation for centripetal force:

\[ a_c = \frac{v_t^2}{r} \]

\[ F_c = ma_c = \frac{mv_t^2}{r} \]

Centripetal force = \( \frac{\text{mass} \times \text{(tangential speed)}^2}{\text{radius of circular path}} \)
Centripetal Force, *continued*

- **Centripetal force** is simply the name given to the net force on an object in uniform circular motion.

- Any type of force or combination of forces can provide this net force.
  - For example, friction between a race car’s tires and a circular track is a centripetal force that keeps the car in a circular path.
  - As another example, gravitational force is a centripetal force that keeps the moon in its orbit.
Centripetal Force, *continued*

- If the centripetal force vanishes, the object stops moving in a circular path.

- A ball that is on the end of a string is whirled in a vertical circular path.
  - If the string breaks at the position shown in (a), the ball will move vertically upward in free fall.
  - If the string breaks at the top of the ball’s path, as in (b), the ball will move along a parabolic path.
Describing a Rotating System

To better understand the motion of a rotating system, consider a car traveling at high speed and approaching an exit ramp that curves to the left.

As the driver makes the sharp left turn, the passenger slides to the right and hits the door.

What causes the passenger to move toward the door?
Describing a Rotating System, continued

As the car enters the ramp and travels along a curved path, the passenger, because of inertia, tends to move along the original straight path.

If a sufficiently large centripetal force acts on the passenger, the person will move along the same curved path that the car does. The origin of the centripetal force is the force of friction between the passenger and the car seat.

If this frictional force is not sufficient, the passenger slides across the seat as the car turns underneath.
Preview

- Objectives
- Gravitational Force
- Applying the Law of Gravitation
Objectives

- **Explain** how Newton’s law of universal gravitation accounts for various phenomena, including satellite and planetary orbits, falling objects, and the tides.

- **Apply** Newton’s law of universal gravitation to solve problems.
Gravitational Force

- Orbiting objects are in free fall.

To see how this idea is true, we can use a thought experiment that Newton developed. Consider a cannon sitting on a high mountaintop.

Each successive cannonball has a greater initial speed, so the horizontal distance that the ball travels increases. If the initial speed is great enough, the curvature of Earth will cause the cannonball to continue falling without ever landing.
Gravitational Force, continued

- The **centripetal force** that holds the planets in orbit is the same force that pulls an apple toward the ground - **gravitational force**.

- **Gravitational force** is the mutual force of attraction between particles of matter.

- Gravitational force depends on the **masses** and on the **distance** between them.
Gravitational Force, continued

Newton developed the following equation to describe quantitatively the magnitude of the gravitational force if distance $r$ separates masses $m_1$ and $m_2$:

Newton's Law of Universal Gravitation

$$F_g = G \frac{m_1 m_2}{r^2}$$

gravitational force = constant $\times \frac{\text{mass 1} \times \text{mass 2}}{(\text{distance between masses})^2}$

The constant $G$, called the constant of universal gravitation, equals $6.673 \times 10^{-11}$ N$\cdot$m$^2$/kg.
Chapter 7

Section 2 Newton’s Law of Universal Gravitation

Newton’s Law of Universal Gravitation

Click below to watch the Visual Concept.

Visual Concept
Gravitational Force, *continued*

- The gravitational forces that two masses exert on each other are always **equal in magnitude** and **opposite in direction**.

- This is an example of **Newton’s third law of motion**.

- One example is the **Earth-moon system**, shown on the next slide.

- As a result of these forces, the moon and Earth each orbit the center of mass of the Earth-moon system. Because Earth has a much greater mass than the moon, this center of mass lies within Earth.
Newton’s Law of Universal Gravitation

\[ F_{Em} = F_c \]

\[ F_{mE} = F_c \]
Applying the Law of Gravitation

- Newton's law of gravitation accounts for ocean tides.

- High and low tides are partly due to the gravitational force exerted on Earth by its moon.

- The tides result from the difference between the gravitational force at Earth's surface and at Earth's center.
Applying the Law of Gravitation, continued

Å Cavendish applied Newton’s law of universal gravitation to find the value of $G$ and Earth’s mass.

Å When two masses, the distance between them, and the gravitational force are known, Newton’s law of universal gravitation can be used to find $G$.

Å Once the value of $G$ is known, the law can be used again to find Earth’s mass.
Gravity is a field force.

Gravitational field strength, $g$, equals $\frac{F_g}{m}$.

The gravitational field, $g$, is a vector with magnitude $g$ that points in the direction of $F_g$.

Gravitational field strength equals free-fall acceleration.
Applying the Law of Gravitation, continued

Å weight = mass × gravitational field strength

Å Because it depends on gravitational field strength, weight changes with location:

\[
\text{weight} = mg
\]

\[
g = \frac{F_g}{m} = \frac{Gmm_E}{mr^2} = \frac{Gm_E}{r^2}
\]

Å On the surface of any planet, the value of \( g \), as well as your weight, will depend on the planet’s mass and radius.
Preview

- Objectives
- Kepler's Laws
- Sample Problem
- Weight and Weightlessness
Objectives

- **Describe** Kepler's laws of planetary motion.

- **Relate** Newton's mathematical analysis of gravitational force to the elliptical planetary orbits proposed by Kepler.

- **Solve** problems involving orbital speed and period.
Kepler’s Laws

Kepler’s laws describe the motion of the planets.

- **First Law:** Each planet travels in an elliptical orbit around the sun, and the sun is at one of the focal points.

- **Second Law:** An imaginary line drawn from the sun to any planet sweeps out equal areas in equal time intervals.

- **Third Law:** The square of a planet’s orbital period ($T^2$) is proportional to the cube of the average distance ($r^3$) between the planet and the sun.
Kepler’s Laws, continued

• Kepler’s laws were developed a generation before Newton’s law of universal gravitation.

• Newton demonstrated that Kepler’s laws are consistent with the law of universal gravitation.

• The fact that Kepler’s laws closely matched observations gave additional support for Newton’s theory of gravitation.
Kepler’s Laws, continued

According to Kepler’s second law, if the time a planet takes to travel the arc on the left ($\Delta t_1$) is equal to the time the planet takes to cover the arc on the right ($\Delta t_2$), then the area $A_1$ is equal to the area $A_2$. Thus, the planet travels faster when it is closer to the sun and slower when it is farther away.
Kepler’s Laws, continued

Kepler’s third law states that $T^2 \propto r^3$.

The constant of proportionality is $4\pi^2/Gm$, where $m$ is the mass of the object being orbited.

So, Kepler’s third law can also be stated as follows:

$$T^2 = \left(\frac{4\pi^2}{Gm}\right)r^3$$
Kepler’s Laws, continued

Kepler’s third law leads to an equation for the period of an object in a circular orbit. The speed of an object in a circular orbit depends on the same factors:

\[ T = 2\pi \sqrt{\frac{r^3}{Gm}} \]
\[ v_t = \sqrt{\frac{Gm}{r}} \]

Note that \( m \) is the mass of the central object that is being orbited. The mass of the planet or satellite that is in orbit does not affect its speed or period.

The mean radius (\( r \)) is the distance between the centers of the two bodies.
### Chapter 7

#### Section 3  Motion in Space

## Planetary Data

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mass (kg)</th>
<th>Mean radius (m)</th>
<th>Mean distance from sun (m)</th>
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<tbody>
<tr>
<td>Earth</td>
<td>$5.97 \times 10^{24}$</td>
<td>$6.38 \times 10^6$</td>
<td>$1.50 \times 10^{11}$</td>
</tr>
<tr>
<td>Earth’s moon</td>
<td>$7.35 \times 10^{22}$</td>
<td>$1.74 \times 10^6$</td>
<td>—</td>
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<tr>
<td>Jupiter</td>
<td>$1.90 \times 10^{27}$</td>
<td>$7.15 \times 10^7$</td>
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<tr>
<td>Mars</td>
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<td>$3.40 \times 10^6$</td>
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<td>Mercury</td>
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<td>Neptune</td>
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<td>Saturn</td>
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<td>Venus</td>
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<td>$6.05 \times 10^6$</td>
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</tr>
</tbody>
</table>
Sample Problem

Period and Speed of an Orbiting Object

Magellan was the first planetary spacecraft to be launched from a space shuttle. During the spacecraft’s fifth orbit around Venus, Magellan traveled at a mean altitude of 361km. If the orbit had been circular, what would Magellan’s period and speed have been?
Sample Problem, continued

1. Define
   Given:
   \[ r_1 = 361 \text{ km} = 3.61 \times 10^5 \text{ m} \]
   Unknown:
   \[ T = ? \quad v_t = ? \]

2. Plan
   Choose an equation or situation: Use the equations for the period and speed of an object in a circular orbit.
   
   \[ T = 2\pi \sqrt{\frac{r^3}{Gm}} \quad v_t = \sqrt{\frac{Gm}{r}} \]
Use Table 1 in the textbook to find the values for the radius ($r_2$) and mass ($m$) of Venus.

$$r_2 = 6.05 \times 10^6 \text{ m} \quad m = 4.87 \times 10^{24} \text{ kg}$$

Find $r$ by adding the distance between the spacecraft and Venus’s surface ($r_1$) to Venus’s radius ($r_2$).

$$r = r_1 + r_2\quad r = 3.61 \times 10^5 \text{ m} + 6.05 \times 10^6 \text{ m} = 6.41 \times 10^6 \text{ m}$$
Sample Problem, continued

3. Calculate

\[ T = 2\pi \sqrt{\frac{r^3}{Gm}} = 2\pi \sqrt{\frac{(6.41 \times 10^6 \text{ m})^3}{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(4.87 \times 10^{24} \text{ kg})}} \]

\[ T = 5.66 \times 10^3 \text{ s} \]

\[ v_t = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(4.87 \times 10^{24} \text{ kg})}{6.41 \times 10^6 \text{ m}}} \]

\[ v_t = 7.12 \times 10^3 \text{ m/s} \]

4. Evaluate

Magellan takes \((5.66 \times 10^3 \text{ s})(1 \text{ min/60 s}) \approx 94 \text{ min to complete one orbit.}\)
Weight and Weightlessness

To learn about apparent weightlessness, imagine that you are in an elevator:

- When the elevator is at rest, the magnitude of the normal force acting on you equals your weight.
- If the elevator were to accelerate downward at 9.81 m/s², you and the elevator would both be in free fall. You have the same weight, but there is no normal force acting on you.
- This situation is called **apparent weightlessness**.
- Astronauts in orbit experience apparent weightlessness.
Weight and Weightlessness

(a) \( a = 0 \)  
\( 600 \text{ N} \)
\( F_n \) (Normal force)  
\( F_g \) (Gravitational force)

(b) \( a \)  
\( 300 \text{ N} \)
\( F_n \) (Normal force)  
\( F_g \) (Gravitational force)

(c) \( a = g \)  
\( 0 \text{ N} \)
\( F_n = 0 \)  
\( F_g \) (Gravitational force)
Chapter 7

Section 4  Torque and Simple Machines

Preview

• Objectives
• Rotational Motion
• The Magnitude of a Torque
• The Sign of a Torque
• Sample Problem
• Simple Machines
Objectives

- **Distinguish** between torque and force.
- **Calculate** the magnitude of a torque on an object.
- **Identify** the six types of simple machines.
- **Calculate** the mechanical advantage of a simple machine.
Rotational Motion

Rotational and translational motion can be analyzed separately.

- For example, when a bowling ball strikes the pins, the pins may spin in the air as they fly backward.
- These pins have both rotational and translational motion.

In this section, we will isolate rotational motion.

In particular, we will explore how to measure the ability of a force to rotate an object.
The Magnitude of a Torque

- **Torque** is a quantity that measures the ability of a force to rotate an object around some axis.
- How easily an object rotates on both how much force is applied and on where the force is applied.
- The perpendicular distance from the axis of rotation to a line drawn along the direction of the force is equal to \( d \sin \theta \) and is called the **lever arm**.

\[
\tau = Fd \sin \theta
\]

**torque = force \times \text{lever arm}**
The Magnitude of a Torque, continued

- The applied force may act at an angle.

- However, the direction of the lever arm \((d \sin \theta)\) is always perpendicular to the direction of the applied force, as shown here.
Torque

Click below to watch the Visual Concept.
In each example, the cat is pushing on the door at the same distance from the axis. To produce the same torque, the cat must apply greater force for smaller angles.
The Sign of a Torque

Å Torque is a **vector** quantity. In this textbook, we will assign each torque a positive or negative sign, depending on the direction the force tends to rotate an object.

Å We will use the convention that the sign of the torque is **positive** if the rotation is **counterclockwise** and **negative** if the rotation is **clockwise**.

**Tip:** To determine the sign of a torque, imagine that the torque is the only one acting on the object and that the object is free to rotate. Visualize the direction that the object would rotate. If more than one force is acting, treat each force separately.
The Sign of a Torque

Click below to watch the Visual Concept.
Sample Problem

Torque

A basketball is being pushed by two players during tip-off. One player exerts an upward force of 15 N at a perpendicular distance of 14 cm from the axis of rotation. The second player applies a downward force of 11 N at a distance of 7.0 cm from the axis of rotation. Find the net torque acting on the ball about its center of mass.
Sample Problem, continued

1. Define

   Given:
   \[ F_1 = 15 \text{ N} \quad F_2 = 11 \text{ N} \]
   \[ d_1 = 0.14 \text{ m} \quad d_2 = 0.070 \text{ m} \]

   Unknown:
   \[ \tau_{net} = ? \]

   Diagram:
Sample Problem, continued

2. Plan

Choose an equation or situation: Apply the definition of torque to each force, and add up the individual torques.

\[ \tau = Fd \]

\[ \tau_{\text{net}} = \tau_1 + \tau_2 = F_1d_1 + F_2d_2 \]

Tip: The factor \( \sin \theta \) is not included in the torque equation because each given distance is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force. In other words, each given distance is the lever arm.
Sample Problem, *continued*

3. Calculate

Substitute the values into the equation and solve: First, determine the torque produced by each force. Use the standard convention for signs.

\[
\tau_1 = F_1d_1 = (15 \text{ N})(0.14 \text{ m}) = 2.1 \text{ N} \cdot \text{m}
\]

\[
\tau_2 = F_2d_2 = (11 \text{ N})(0.070 \text{ m}) = 0.77 \text{ N} \cdot \text{m}
\]

\[
\tau_{\text{net}} = \tau_1 + \tau_2 = 2.1 \text{ N} \cdot \text{m} + 0.77 \text{ N} \cdot \text{m}
\]

\[
\tau_{\text{net}} = 2.9 \text{ N} \cdot \text{m}
\]

4. Evaluate

The net torque is negative, so the ball rotates in a clockwise direction.
A machine is any device that transmits or modifies force, usually by changing the force applied to an object.

All machines are combinations or modifications of six fundamental types of machines, called simple machines.

These six simple machines are the lever, pulley, inclined plane, wheel and axle, wedge, and screw.
Simple Machines

Click below to watch the Visual Concept.
Simple Machines, continued

Because the purpose of a simple machine is to change the direction or magnitude of an input force, a useful way of characterizing a simple machine is to compare the output and input force.

This ratio is called mechanical advantage.

If friction is disregarded, mechanical advantage can also be expressed in terms of input and output distance.

\[
MA = \frac{F_{out}}{F_{in}} = \frac{d_{in}}{d_{out}}
\]
Simple Machines, continued

The diagrams show two examples of a trunk being loaded onto a truck.

- In the first example, a force $F_1$ of 360 N moves the trunk through a distance $d_1$ of 1.0 m. This requires 360 N\(\cdot\)m of work.

- In the second example, a lesser force $F_2$ of only 120 N would be needed (ignoring friction), but the trunk must be pushed a greater distance $d_2$ of 3.0 m. This also requires 360 N\(\cdot\)m of work.
The simple machines we have considered so far are ideal, frictionless machines.

Real machines, however, are not frictionless. Some of the input energy is dissipated as sound or heat.

The efficiency of a machine is the ratio of useful work output to work input.

\[ \text{eff} = \frac{W_{\text{out}}}{W_{\text{in}}} \]

- The efficiency of an ideal (frictionless) machine is 1, or 100 percent.
- The efficiency of real machines is always less than 1.
Mechanical Efficiency

Click below to watch the Visual Concept.